

LASTNAME :

STUDENT ID :

FIRSTNAME :

# Final Exam

1<sup>st</sup> session

Saturday 9 June 2012 – aud. H.2215

## Indications

Please follow these indications:

1. The exam lasts 2.5 hours in total but was designed to be answered in 2 hours.
2. Please verify that your document contains exactly 8 pages.
3. You are allowed to come with a 5-page recto-verso handwritten cheat sheet, i.e. 10 pages in total.
4. Each question is worth one point unless otherwise stated, i.e. there are 22 points in total, including two “bonus” (or facultative) questions of 3 points each.
5. Please write your first name and last name on the first page.
6. Good work!!!

## Problems

### P1 Short questions

- Q1** It is July 16. A company has a portfolio of stocks worth \$100 million. The beta of the portfolio is 1.2. The company would like to use the CME December futures contract on the S&P 500 (the beta of the futures contract is 1) to change the beta of the portfolio to 0.5 during the period July 16 to November 16. The index is currently 1'000, and each contract is on \$250 times the index. What position should the company take?

- Q2** Is a portfolio with a large negative gamma but a delta of 0 risky? Why? Explain in detail.

**Q3** Is the total risk exposure on a pair of offsetting interest rate swaps higher or lower than for a pair of offsetting currency swap? Explain carefully.

**Q4** A portfolio consists of a one year zero-coupon bond with a face value of €100 and a 2-year coupon bond bearing an interest of 5% with a principal value of €100. The current yield on all bonds is 3% per annum. Compute the change in the value of this portfolio for a 1% per annum increase in yield (using duration and convexity, all rates are continuous).

**Q5** What is the advantage of using partial durations and not only global durations, for a portfolio or a bank balance sheet for example? (don't provide just the definition or the trivial comparison but rather try to show the added value or the kind of advantage in terms of risk monitoring when we have partial durations available)

**Q6** Please check the right box (true or false) for each statement. (2 points in total)

True	False	Statement
<input type="checkbox"/>	<input type="checkbox"/>	The VaR concept bears the problem that it relies on the hypothesis of normal distribution of the underlying variable's returns.
<input type="checkbox"/>	<input type="checkbox"/>	The minimum variance-hedge ratio is a useful technique to compute the amount of hedging futures when there is a basis risk involved.
<input type="checkbox"/>	<input type="checkbox"/>	The exposure of a naked short put position is more important than the exposure with a naked short call position.
<input type="checkbox"/>	<input type="checkbox"/>	In a CDO, all tranches have the same risk profile.

**P2 Volatilities**

**Q7** Suppose that the silver spot price closed yesterday at €29 and its volatility was estimated at 2% per day. The today's closing price is €30. Please update your volatility estimation in the context of the EWMA model with  $\lambda=0.94$ .

**Q8** Do the same as in question Q7 but assuming this time a GARCH(1,1) model with  $\omega = 0.000006$ ,  $\alpha = 0.03$  and  $\beta = 0.92$ .

**P3 Credit risk in a portfolio, and credit derivatives**

**Q9** Suppose that a bank has made a large number loans of a certain type. The total amount lent is \$100 million. The one-year probability of default on each loan is 1.2% and the loss when a default occurs is 70% of the amount owed. The bank uses the Vasicek copula. The copula correlation parameter is 0.3. Estimate the loss on the portfolio that is not expected to be exceeded with a probability of 99%. (2 points)

**Q10** A company has issued 1- and 2-year bonds with a coupon of 4% payable annually. The yields on the bonds (expressed with continuous compounding) are 4.5% and 5%, respectively. Risk free rates are 3.5% with continuous compounding for all maturities. The recovery rate is 40%. Defaults can take place halfway through each year. Compute the risk-neutral default rates for year 1 and 2. (2 points)

**Q11** Suppose that the risk-free zero curve is flat at 6% per annum with continuous compounding and that defaults can occur at times 0.5 and 1.5 years in a 2-year plain vanilla credit default swap with annual payments. Suppose the recovery rate is 40% and the unconditional probabilities of default (as seen at time zero) are 1% at time 0.5 and 1.5% at time 1.5 years. What is the CDS spread? (2 points)

**P4 Interest-rate risk measurement**

In the table below, you will find data from the risk management department concerning the change in value of your bond portfolio for some moves of the yield curve (in bp).

Year	Move (bp)	Change (million €)
1	3	-0.84
2	5	-0.6
3	2	0.1
5	2	0.5
7	1	0.3
10	1	-0.2

**Q12** Can you find the values of partial durations for those years (the total value of your portfolio is € 10 million)?

**Q13** What's the total duration of your portfolio (you invest in bonds with maturities  $\leq 10$  years)?

**Q14** Using the table below, compute the delta for the first and second principal components. How can you interpret those numbers?

Factor loadings for US Treasury data		
	PC1	PC2
3m	0,21	-0,57
6m	0,26	-0,49
12m	0,32	-0,32
2y	0,35	-0,1
3y	0,36	0,02
4y	0,36	0,14
5y	0,36	0,17
7y	0,34	0,27
10y	0,31	0,3
30y	0,25	0,33

**P5 VaR**

**Q15** A portfolio consists of options on GM and Google. The options on GM have a delta of 1'000, and the options on Google have a delta 5'000. The GM share price is \$30, and the Google share price is \$400. Assuming the daily volatility of GM is 1% and the daily volatility of Google is 3% and the correlation between the daily change is -0.1. Compute the 1-day 99% VaR of your portfolio.

**Q16** Suppose that we back test the previous model 100 days of data and we observe 3 exceptions. Should we reject the model at the 5% confidence level? Use a one-tailed test.

Suppose that each of two investments has a 3% chance of loss of \$1 million, a 6% chance of a loss of \$0.5 million, and a 91% chance of a profit of \$0.1 million. They are independent of each other. [This applies to questions 17 and 18.]

**Q17** What is the VaR for a portfolio consisting of the two investments when the confidence level is 95%?

**Q18** What is the expected shortfall for a portfolio consisting of the two investments when the confidence level is 95%?

**N(x) & N(-x)=1-N(x)**

	<b>0.000</b>	<b>0.005</b>	<b>0.010</b>	<b>0.015</b>	<b>0.020</b>	<b>0.025</b>	<b>0.030</b>	<b>0.035</b>	<b>0.040</b>	<b>0.045</b>	<b>0.050</b>	<b>0.055</b>	<b>0.060</b>	<b>0.065</b>	<b>0.070</b>	<b>0.075</b>	<b>0.080</b>	<b>0.085</b>	<b>0.090</b>	<b>0.095</b>
<b>0.0</b>	0.5000	0.5020	0.5040	0.5060	0.5080	0.5100	0.5120	0.5140	0.5160	0.5179	0.5199	0.5219	0.5239	0.5259	0.5279	0.5299	0.5319	0.5339	0.5359	0.5378
<b>0.1</b>	0.5398	0.5418	0.5438	0.5458	0.5478	0.5497	0.5517	0.5537	0.5557	0.5576	0.5596	0.5616	0.5636	0.5655	0.5675	0.5695	0.5714	0.5734	0.5753	0.5773
<b>0.2</b>	0.5793	0.5812	0.5832	0.5851	0.5871	0.5890	0.5910	0.5929	0.5948	0.5968	0.5987	0.6006	0.6026	0.6045	0.6064	0.6083	0.6103	0.6122	0.6141	0.6160
<b>0.3</b>	0.6179	0.6198	0.6217	0.6236	0.6255	0.6274	0.6293	0.6312	0.6331	0.6350	0.6368	0.6387	0.6406	0.6424	0.6443	0.6462	0.6480	0.6499	0.6517	0.6536
<b>0.4</b>	0.6554	0.6573	0.6591	0.6609	0.6628	0.6646	0.6664	0.6682	0.6700	0.6718	0.6736	0.6754	0.6772	0.6790	0.6808	0.6826	0.6844	0.6862	0.6879	0.6897
<b>0.5</b>	0.6915	0.6932	0.6950	0.6967	0.6985	0.7002	0.7019	0.7037	0.7054	0.7071	0.7088	0.7106	0.7123	0.7140	0.7157	0.7174	0.7190	0.7207	0.7224	0.7241
<b>0.6</b>	0.7257	0.7274	0.7291	0.7307	0.7324	0.7340	0.7357	0.7373	0.7389	0.7405	0.7422	0.7438	0.7454	0.7470	0.7486	0.7502	0.7517	0.7533	0.7549	0.7565
<b>0.7</b>	0.7580	0.7596	0.7611	0.7627	0.7642	0.7658	0.7673	0.7688	0.7704	0.7719	0.7734	0.7749	0.7764	0.7779	0.7794	0.7808	0.7823	0.7838	0.7852	0.7867
<b>0.8</b>	0.7881	0.7896	0.7910	0.7925	0.7939	0.7953	0.7967	0.7981	0.7995	0.8009	0.8023	0.8037	0.8051	0.8065	0.8078	0.8092	0.8106	0.8119	0.8133	0.8146
<b>0.9</b>	0.8159	0.8173	0.8186	0.8199	0.8212	0.8225	0.8238	0.8251	0.8264	0.8277	0.8289	0.8302	0.8315	0.8327	0.8340	0.8352	0.8365	0.8377	0.8389	0.8401
<b>1.0</b>	0.8413	0.8426	0.8438	0.8449	0.8461	0.8473	0.8485	0.8497	0.8508	0.8520	0.8531	0.8543	0.8554	0.8566	0.8577	0.8588	0.8599	0.8610	0.8621	0.8632
<b>1.1</b>	0.8643	0.8654	0.8665	0.8676	0.8686	0.8697	0.8708	0.8718	0.8729	0.8739	0.8749	0.8760	0.8770	0.8780	0.8790	0.8800	0.8810	0.8820	0.8830	0.8840
<b>1.2</b>	0.8849	0.8859	0.8869	0.8878	0.8888	0.8897	0.8907	0.8916	0.8925	0.8934	0.8944	0.8953	0.8962	0.8971	0.8980	0.8988	0.8997	0.9006	0.9015	0.9023
<b>1.3</b>	0.9032	0.9041	0.9049	0.9057	0.9066	0.9074	0.9082	0.9091	0.9099	0.9107	0.9115	0.9123	0.9131	0.9139	0.9147	0.9154	0.9162	0.9170	0.9177	0.9185
<b>1.4</b>	0.9192	0.9200	0.9207	0.9215	0.9222	0.9229	0.9236	0.9244	0.9251	0.9258	0.9265	0.9272	0.9279	0.9285	0.9292	0.9299	0.9306	0.9312	0.9319	0.9325
<b>1.5</b>	0.9332	0.9338	0.9345	0.9351	0.9357	0.9364	0.9370	0.9376	0.9382	0.9388	0.9394	0.9400	0.9406	0.9412	0.9418	0.9424	0.9429	0.9435	0.9441	0.9446
<b>1.6</b>	0.9452	0.9458	0.9463	0.9468	0.9474	0.9479	0.9484	0.9490	0.9495	0.9500	0.9505	0.9510	0.9515	0.9520	0.9525	0.9530	0.9535	0.9540	0.9545	0.9550
<b>1.7</b>	0.9554	0.9559	0.9564	0.9568	0.9573	0.9577	0.9582	0.9586	0.9591	0.9595	0.9599	0.9604	0.9608	0.9612	0.9616	0.9621	0.9625	0.9629	0.9633	0.9637
<b>1.8</b>	0.9641	0.9645	0.9649	0.9652	0.9656	0.9660	0.9664	0.9667	0.9671	0.9675	0.9678	0.9682	0.9686	0.9689	0.9693	0.9696	0.9699	0.9703	0.9706	0.9710
<b>1.9</b>	0.9713	0.9716	0.9719	0.9723	0.9726	0.9729	0.9732	0.9735	0.9738	0.9741	0.9744	0.9747	0.9750	0.9753	0.9756	0.9759	0.9761	0.9764	0.9767	0.9770
<b>2.0</b>	0.9772	0.9775	0.9778	0.9780	0.9783	0.9786	0.9788	0.9791	0.9793	0.9796	0.9798	0.9801	0.9803	0.9805	0.9808	0.9810	0.9812	0.9815	0.9817	0.9819
<b>2.1</b>	0.9821	0.9824	0.9826	0.9828	0.9830	0.9832	0.9834	0.9836	0.9838	0.9840	0.9842	0.9844	0.9846	0.9848	0.9850	0.9852	0.9854	0.9856	0.9857	0.9859
<b>2.2</b>	0.9861	0.9863	0.9864	0.9866	0.9868	0.9870	0.9871	0.9873	0.9875	0.9876	0.9878	0.9879	0.9881	0.9882	0.9884	0.9885	0.9887	0.9888	0.9890	0.9891
<b>2.3</b>	0.9893	0.9894	0.9896	0.9897	0.9898	0.9900	0.9901	0.9902	0.9904	0.9905	0.9906	0.9907	0.9909	0.9910	0.9911	0.9912	0.9913	0.9915	0.9916	0.9917
<b>2.4</b>	0.9918	0.9919	0.9920	0.9921	0.9922	0.9923	0.9925	0.9926	0.9927	0.9928	0.9929	0.9930	0.9931	0.9931	0.9932	0.9933	0.9934	0.9935	0.9936	0.9937
<b>2.5</b>	0.9938	0.9939	0.9940	0.9941	0.9942	0.9943	0.9944	0.9945	0.9945	0.9946	0.9946	0.9947	0.9948	0.9948	0.9949	0.9950	0.9951	0.9951	0.9952	0.9953
<b>2.6</b>	0.9953	0.9954	0.9955	0.9955	0.9956	0.9957	0.9957	0.9958	0.9959	0.9959	0.9960	0.9960	0.9961	0.9962	0.9962	0.9963	0.9963	0.9964	0.9964	0.9965
<b>2.7</b>	0.9965	0.9966	0.9966	0.9967	0.9967	0.9968	0.9968	0.9969	0.9969	0.9970	0.9970	0.9971	0.9971	0.9972	0.9972	0.9972	0.9973	0.9973	0.9974	0.9974
<b>2.8</b>	0.9974	0.9975	0.9975	0.9976	0.9976	0.9976	0.9977	0.9977	0.9977	0.9978	0.9978	0.9978	0.9979	0.9979	0.9979	0.9980	0.9980	0.9980	0.9981	0.9981
<b>2.9</b>	0.9981	0.9982	0.9982	0.9982	0.9982	0.9983	0.9983	0.9983	0.9984	0.9984	0.9984	0.9985	0.9985	0.9985	0.9985	0.9986	0.9986	0.9986	0.9986	0.9986
<b>3.0</b>	0.9987	0.9987	0.9987	0.9987	0.9987	0.9988	0.9988	0.9988	0.9988	0.9988	0.9989	0.9989	0.9989	0.9989	0.9989	0.9989	0.9990	0.9990	0.9990	0.9990
<b>3.1</b>	0.9990	0.9990	0.9991	0.9991	0.9991	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993	0.9993	0.9993	0.9993
<b>3.2</b>	0.9993	0.9993	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995	0.9995	0.9995	0.9995	0.9995
<b>3.3</b>	0.9995	0.9995	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997	0.9997	0.9997
<b>3.4</b>	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998	0.9998	0.9998
<b>3.5</b>	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
<b>3.6</b>	0.9998	0.9998	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
<b>3.7</b>	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
<b>3.8</b>	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
<b>3.9</b>	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
<b>4.0</b>	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Binomial probability distribution  $C_{n,r} p^r (1-p)^{(n-r)}$

n	r	p																			
		0.01	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
5	0	0.951	0.774	0.590	0.444	0.328	0.237	0.168	0.116	0.078	0.050	0.031	0.018	0.010	0.005	0.002	0.001	0.000	0.000	0.000	0.000
5	1	0.048	0.204	0.328	0.392	0.410	0.396	0.360	0.312	0.259	0.206	0.156	0.113	0.077	0.049	0.028	0.015	0.006	0.002	0.000	0.000
5	2	0.001	0.021	0.073	0.138	0.205	0.264	0.309	0.336	0.346	0.337	0.313	0.276	0.230	0.181	0.132	0.088	0.051	0.024	0.008	0.001
5	3	0.000	0.001	0.008	0.024	0.051	0.088	0.132	0.181	0.230	0.276	0.313	0.337	0.346	0.336	0.309	0.264	0.205	0.138	0.073	0.021
5	4	0.000	0.000	0.000	0.002	0.006	0.015	0.028	0.049	0.077	0.113	0.156	0.206	0.259	0.312	0.360	0.396	0.410	0.392	0.328	0.204
5	5	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.005	0.010	0.018	0.031	0.050	0.078	0.116	0.168	0.237	0.328	0.444	0.590	0.774
10	0	0.904	0.599	0.349	0.197	0.107	0.056	0.028	0.013	0.006	0.003	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10	1	0.091	0.315	0.387	0.347	0.268	0.188	0.121	0.072	0.040	0.021	0.010	0.004	0.002	0.001	0.000	0.000	0.000	0.000	0.000	0.000
10	2	0.004	0.075	0.194	0.276	0.302	0.282	0.233	0.176	0.121	0.076	0.044	0.023	0.011	0.004	0.001	0.000	0.000	0.000	0.000	0.000
10	3	0.000	0.010	0.057	0.130	0.201	0.250	0.267	0.252	0.215	0.166	0.117	0.075	0.042	0.021	0.009	0.003	0.001	0.000	0.000	0.000
10	4	0.000	0.001	0.011	0.040	0.088	0.146	0.200	0.238	0.251	0.238	0.205	0.160	0.111	0.069	0.037	0.016	0.006	0.001	0.000	0.000
10	5	0.000	0.000	0.001	0.008	0.026	0.058	0.103	0.154	0.201	0.234	0.246	0.234	0.201	0.154	0.103	0.058	0.026	0.008	0.001	0.000
10	6	0.000	0.000	0.000	0.001	0.006	0.016	0.037	0.069	0.111	0.160	0.205	0.238	0.251	0.238	0.200	0.146	0.088	0.040	0.011	0.001
10	7	0.000	0.000	0.000	0.000	0.001	0.003	0.009	0.021	0.042	0.075	0.117	0.166	0.215	0.252	0.267	0.250	0.201	0.130	0.057	0.010
10	8	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.004	0.011	0.023	0.044	0.076	0.121	0.176	0.233	0.282	0.302	0.276	0.194	0.075
10	9	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.004	0.010	0.021	0.040	0.072	0.121	0.188	0.268	0.347	0.387	0.315
10	10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.003	0.006	0.013	0.028	0.056	0.107	0.197	0.349	0.599
20	0	0.818	0.358	0.122	0.039	0.012	0.003	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
20	1	0.165	0.377	0.270	0.137	0.058	0.021	0.007	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
20	2	0.016	0.189	0.285	0.229	0.137	0.067	0.028	0.010	0.003	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
20	3	0.001	0.060	0.190	0.243	0.205	0.134	0.072	0.032	0.012	0.004	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
20	4	0.000	0.013	0.090	0.182	0.218	0.190	0.130	0.074	0.035	0.014	0.005	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
20	5	0.000	0.002	0.032	0.103	0.175	0.202	0.179	0.127	0.075	0.036	0.015	0.005	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
20	6	0.000	0.000	0.009	0.045	0.109	0.169	0.192	0.171	0.124	0.075	0.037	0.015	0.005	0.001	0.000	0.000	0.000	0.000	0.000	0.000
20	7	0.000	0.000	0.002	0.016	0.055	0.112	0.164	0.184	0.166	0.122	0.074	0.037	0.015	0.004	0.001	0.000	0.000	0.000	0.000	0.000
20	8	0.000	0.000	0.000	0.005	0.022	0.061	0.114	0.161	0.180	0.162	0.120	0.073	0.035	0.014	0.004	0.001	0.000	0.000	0.000	0.000
20	9	0.000	0.000	0.000	0.001	0.007	0.027	0.065	0.116	0.160	0.177	0.160	0.119	0.071	0.034	0.012	0.003	0.000	0.000	0.000	0.000
20	10	0.000	0.000	0.000	0.000	0.002	0.010	0.031	0.069	0.117	0.159	0.176	0.159	0.117	0.069	0.031	0.010	0.002	0.000	0.000	0.000
20	11	0.000	0.000	0.000	0.000	0.000	0.003	0.012	0.034	0.071	0.119	0.160	0.177	0.160	0.116	0.065	0.027	0.007	0.001	0.000	0.000
20	12	0.000	0.000	0.000	0.000	0.000	0.001	0.004	0.014	0.035	0.073	0.120	0.162	0.180	0.161	0.114	0.061	0.022	0.005	0.000	0.000
20	13	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.004	0.015	0.037	0.074	0.122	0.166	0.184	0.164	0.112	0.055	0.016	0.002	0.000
20	14	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.005	0.015	0.037	0.075	0.124	0.171	0.192	0.169	0.109	0.045	0.009	0.000
20	15	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.005	0.015	0.036	0.075	0.127	0.179	0.202	0.175	0.103	0.032	0.002
20	16	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.005	0.014	0.035	0.074	0.130	0.190	0.218	0.182	0.090	0.013
20	17	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.004	0.012	0.032	0.072	0.134	0.205	0.243	0.190	0.060	0.000
20	18	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.003	0.010	0.028	0.067	0.137	0.229	0.285	0.189	0.000
20	19	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.007	0.021	0.058	0.137	0.270	0.377
20	20	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.003	0.012	0.039	0.122	0.358	0.818